

Model building on the non-factorisable type IIA $T^6/(\mathbb{Z}_4 \times \Omega\mathcal{R})$ orientifold

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We construct global semi-realistic supersymmetric models with intersecting D6-branes on the non-factorisable orientifold $T^6/(\mathbb{Z}_4 \times \Omega\mathcal{R})$. The non-factorisable structure gives rise to differences compared to the factorisable case: additional conditions for the three-cycles to be Lagrangian and extra constraints on the wrapping numbers for building fractional cycles.

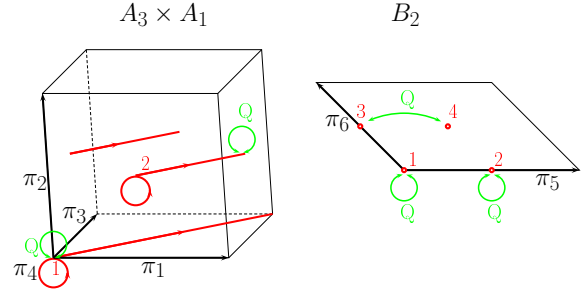


Figure 1 T^6/\mathbb{Z}_4 -orbifold on the $A_3 \times A_1 \times B_2$ -lattice and its \mathbb{Z}_2 -fixed lines (in red).

1 Geometry of the T^6/\mathbb{Z}_4 -orbifold

So far only factorisable orbifolds have been considered for model building [1] and moduli stabilisation [2] on $T^6/(\mathbb{Z}_4 \times \Omega\mathcal{R})$. The models on the non-factorisable ones were to our best knowledge discussed only in [3] with the restriction that the D6-branes are on the top of the O6-planes. In our case, we construct models with arbitrary D6-branes on $T^6/(\mathbb{Z}_4 \times \Omega\mathcal{R})$ with the torus lattice $A_3 \times A_1 \times B_2$, see figure 1.

The Coxeter element Q acts on this root lattice spanned by the simple roots $\{e_i\}_{i=1,\dots,6}$ as

$$Qe_1 = e_2, \quad Qe_2 := e_3, \quad Qe_3 := -e_1 - e_2 - e_3,$$

$$Qe_4 = -e_4, \quad Qe_5 := e_5 + 2e_6, \quad Qe_6 := -e_5 - e_6.$$

This action preserves $\mathcal{N} = 2$ supersymmetry in four dimensions and fixes the metric of the six-torus up to four angles and three radii [4]. The Hodge numbers of this T^6/\mathbb{Z}_4 -orbifold are $h_{21} = h_{21}^{\text{untw}} + h_{21}^{\mathbb{Z}_2} = 1 + 2$ and $h_{11} = h_{11}^{\text{untw}} + h_{11}^{\mathbb{Z}_4} + h_{11}^{\mathbb{Z}_2} = 5 + 16 + 6$.

We first consider the three-cycles inherited from the six-torus and denote a one-cycle in the direction of the lattice vector e_i by π_i . The idea is to describe the toroidal three-cycles in a similar way as in the factorisable case, namely by wrapping numbers along the fundamental cycles, so that each three-cycle can be written as a product of three one-cycles. But in contrast to the factorisable case, we also have cycles like π_{135} which wrap a two-cycle on the A_3 -torus and a one-cycle on the B_2 -torus.

Thus, we can make the ansatz that any toroidal three-cycle

can be described by ten wrapping numbers s.t.

$$\pi^{\text{torus}} := \bigwedge_{i=1}^2 (m^i \pi_1 + n^i \pi_2 + p^i \pi_3 + q^i \pi_4) \wedge (m^3 \pi_5 + n^3 \pi_6). \quad (1)$$

By taking orbits of the Q -action ($\tilde{Q} := \sum_{n=0}^3 Q^n$), a basis of \mathbb{Z}_4 -invariant *bulk* three-cycles is given by

$$\gamma_1 := -\tilde{Q}\pi_{136} \quad \gamma_2 := -\tilde{Q}\pi_{125}, \quad \tilde{\gamma}_1 := \tilde{Q}\pi_{146}, \quad \tilde{\gamma}_2 := \tilde{Q}\pi_{246}.$$

By means of the \mathbb{Z}_2 -invariant one-cycles $\pi_1 + \pi_3$ and π_4 , we can construct the \mathbb{Z}_4 -invariant *exceptional* three-cycles

$$\begin{aligned} \gamma_3 &:= (e_{13} - e_{14}) \wedge (\pi_1 + \pi_3), & \tilde{\gamma}_3 &:= (e_{13} - e_{14}) \wedge \pi_4, \\ \gamma_4 &:= (e_{23} - e_{24}) \wedge (\pi_1 + \pi_3), & \tilde{\gamma}_4 &:= (e_{23} - e_{24}) \wedge \pi_4, \end{aligned}$$

where e_{ij} are exceptional two-cycles with the topology of S^2 arising from the \mathbb{Z}_2 -orbifold singularities by blowing

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up. The non-vanishing intersection numbers between the bulk and/or exceptional three-cycles are given by

$$\gamma_i \circ \bar{\gamma}_j = -2\delta_{ij} \quad i, j = 1, 2, \quad \gamma_m \circ \bar{\gamma}_n = 2\delta_{mn} \quad m, n = 3, 4.$$

Since the intersection form of the γ_i 's, and $\bar{\gamma}_i$'s ($i = 1, 2, 3, 4$) is not unimodular, these three-cycles do not form an integral basis. Thus, we have to consider fractional three-cycles, which can consist of half a bulk cycle and simultaneously of half an exceptional cycle.

If in the case of the factorisable torus every bulk cycle passing through \mathbb{Z}_2 -invariant points in $(T^2)^3$ can split into fractional cycles, here this does not hold true any more, so that we have to consider

$$\pi \text{ is fractional} \iff Q^2 \pi^{\text{torus}} = \pi^{\text{torus}},$$

which leads to some constraints on the wrapping numbers (m^i, n^i, p^i, q^i) . Furthermore, we find that the fractional cycles form the $F_4 \oplus F_4$ lattice, the basis of which gives rise to the unimodular basis of $H_3(T^6/\mathbb{Z}_4; \mathbb{Z})$.

2 IIA Orientifolds on the \mathbb{Z}_4 -orbifold

2.1 Supersymmetry

The orientifold projection $\Omega\mathcal{R}$, where \mathcal{R} is an anti-holomorphic involution $z_i \rightarrow e^{i\theta_i} \bar{z}_i$, breaks supersymmetry to $\mathcal{N} = 1$. The O6-planes are given by the fixed point loci of the involution. Their presence enables the fulfilment of the RR-tadpole cancellation conditions, which allows for the existence of global models.

The T^6/\mathbb{Z}_4 -orbifold has only one continuous complex structure \mathcal{U} inherited from the six-torus [4]. The crystallographic action of \mathcal{R} on the root lattice then fixes $\text{Re}(\mathcal{U}) = 0, \frac{1}{2}$ (for $\theta_i = 0$).

We are interested in supersymmetric D6-branes. This implies that all three-cycles wrapped by them have to be *special Lagrangian* (sLag):

$$J|_{\pi^{\text{torus}}} = 0, \quad \text{Im}(\Omega_3)|_{\pi^{\text{torus}}} = 0, \quad \text{Re}(\Omega_3)|_{\pi^{\text{torus}}} > 0,$$

where Ω_3 is the holomorphic volume-form and J the Kähler-form. The anti-symmetry of the Kähler form under the involution ($\mathcal{R}J = -J$) fixes one of the angle-moduli. Contrary to the factorisable case, where the three-cycles satisfy the *Lag* condition ($J|_{\pi^{\text{torus}}} = 0$) automatically, in the present non-factorisable case the *Lag* condition gives rise to four constraints on the wrapping numbers of the toroidal three-cycle of eq (1). Half of them can be fulfilled by fixing an angular modulus and the remaining constraints are equivalent to the fractional ones, i.e. any *Lag* three-cycle passing through the \mathbb{Z}_2 -fixed points is \mathbb{Z}_2 -invariant and thus can be made fractional, and vice versa.

2.2 Supersymmetric Pati-Salam models

The involution \mathcal{R} (for $\theta_i = 0$) gives rise to two possible lattice configurations with $\text{Re}(\mathcal{U}) = 0$ or $\frac{1}{2}$. For the lattice with $\text{Re}(\mathcal{U}) = \frac{1}{2}$, there exist some *local* Pati-Salam (PS) models with three generations, but these come with chiral matter states transforming in the anti-symmetric representation of the gauge group of the a -stack. We also found *global* PS models with two and four generations for both lattices. The exemplary chiral spectrum of a *global* four-generation PS model is displayed in the table below:

Chiral spectrum of a global Pati-Salam model with four generations (for $\text{Re}(\mathcal{U}) = 0$)	
sector	$SU(4)_a \times [USp \text{ or } SO](2)_b \times [USp \text{ or } SO](2)_c \times [USp \text{ or } SO](4)_d \times U(1)_a$
$ab = ab'$	$4 \times (4, \bar{2}, 1, 1)_{+1}$
$ac = ac'$	$4 \times (\bar{4}, 1, 2, 1)_{-1}$

3 Results and Outlook

We studied the geometry of the non-factorisable $T^6/(\mathbb{Z}_4 \times \Omega\mathcal{R})$ -orientifold and verified the analogies and differences to the factorisable case. We found that global supersymmetric PS models with two and four generations are possible. We aim at extending the search for global three-generation PS models by the analysis of different choices of the involution \mathcal{R} , and we will classify the type of symmetry enhancement $[USp \text{ or } SO]$ based on conformal field theory methods, see e.g. [5]. Last but not least, studying non-factorisable tori is motivated by moduli stabilisation involving closed string background fluxes, which we plan to address in the future.

Acknowledgements. This work is partially supported by the Cluster of Excellence PRISMA DGF no. EXC 1098, the DFG research grant HO 4166/2-1 and the GRK 1581.

Key words. Model building, orientifolds, intersecting D-branes

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